Cubic bulk as benchmark for 3D modelling of superconductors under slowly varying magnetic fields

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Model parameters

- Bulk dimensions: $a \times a \times a = 10 \times 10 \times 10$ mm.
- Critical current density J_c=10^8 A/m ^{2}.
- Sinusoidal applied magnetic field amplitude 200 mT in z direction.
- Frequency 50 Hz.
- Power law n-factor 100.

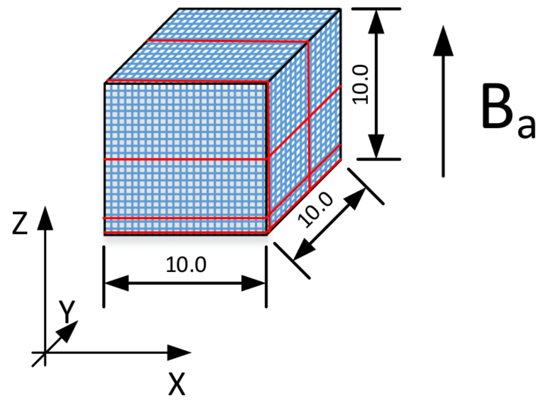

Figure 1: The cubic bulk sample allows benchmarking of the 3D models.

Numerical methods

1. **Minimum electro-magnetic entropy production (MEMEP)** in three dimensions with novel functional [1–5]. The computing time for the initial magnetization curve is around 2 and 8 hours for tolerance of $10^{-3}J_c$ and $10^{-5}J_c$, respectively, and a computer with Intel(R) Core(TM) i7-4771 CPU@3.50GHz, 8 GB RAM, Linux Ubuntu 64 bit.

2. **H-formulation of Maxwell's equations with finite-element method** in three dimensions implemented in *Comsol Multiphysics* [6, 7]. The computing time for the initial magnetization curve is around 11 hours on a computer with Intel(R) Core(TM) i7-4960X CPU@3.60 GHz, 64 GB RAM, Windows 7 64 bit.

Numerical specifications

1. **Minimum electro-magnetic entropy production (MEMEP)**

   - Tolerance for **J**: $10^{-5}J_c$ (for $10^{-5}J_c$ is necessary for accurate **J** plots, $10^{-3}J_c$ is sufficient for accurate magnetization).
   - Mesh: cubic regular (for the results presented in this file). Numerical methods not supporting cubic mesh may use regular tetrahedral elements.
   - Total number of cells $41 \times 41 \times 41$.
   - Total number of degrees of freedom: 211806 (unknown variable **T**, stored at edges).

2. **H-formulation of Maxwell's equations with finite-element method**

   - Mesh: tetrahedral elements for the superconducting cube region, pyramid elements for the air.
   - Total number of degrees of freedom: 606723 (unknown variable **H**, stored at edges).
   - Relative tolerance 0.1

Results

- Current density **J** at cross-sections: J_y at y=5 mm, J_z at y=0.12 mm and z=1.1 mm, $|\mathbf{J}|$ at z=0.12 mm and z=5 mm.

- Initial magnetization curve.

- In each figure, the order of presentations is: MEMEP, **H**-formulation.

References

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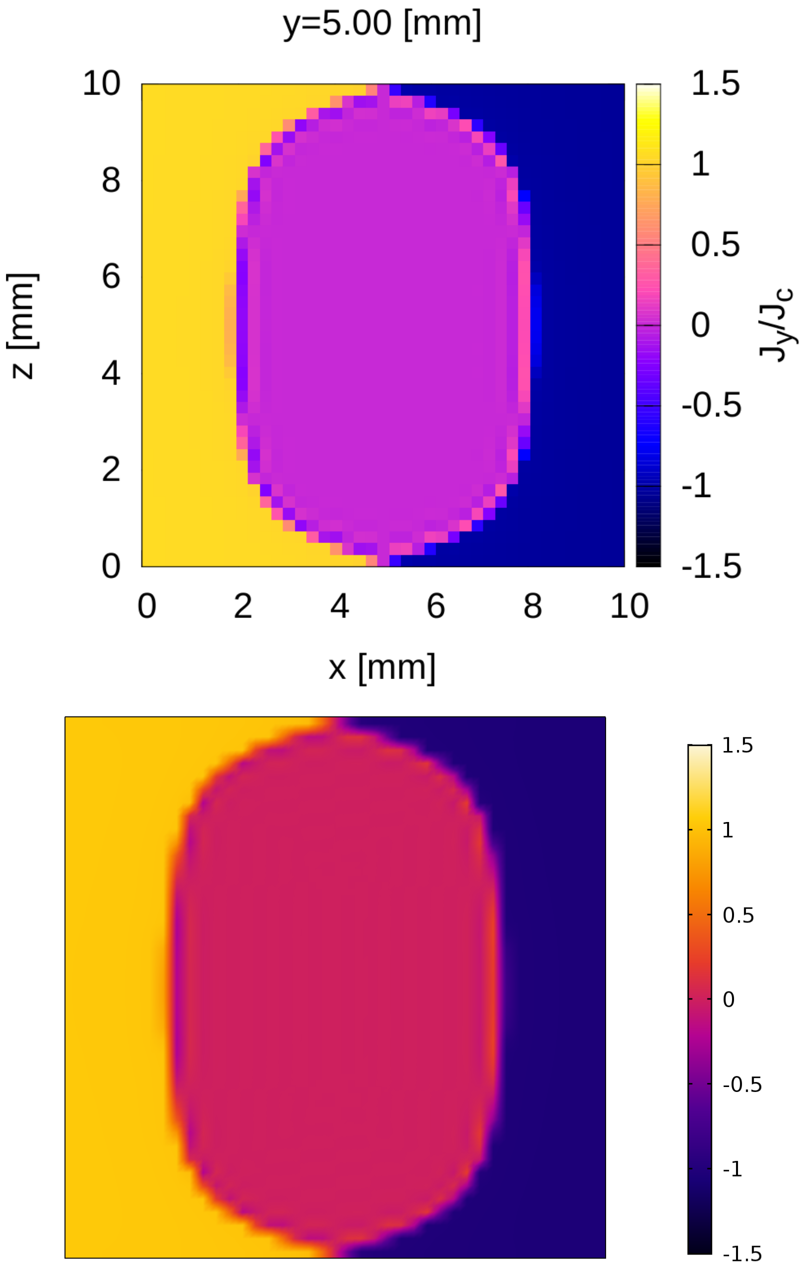

Figure 2: Component of current density J_y at plane y=5 mm.

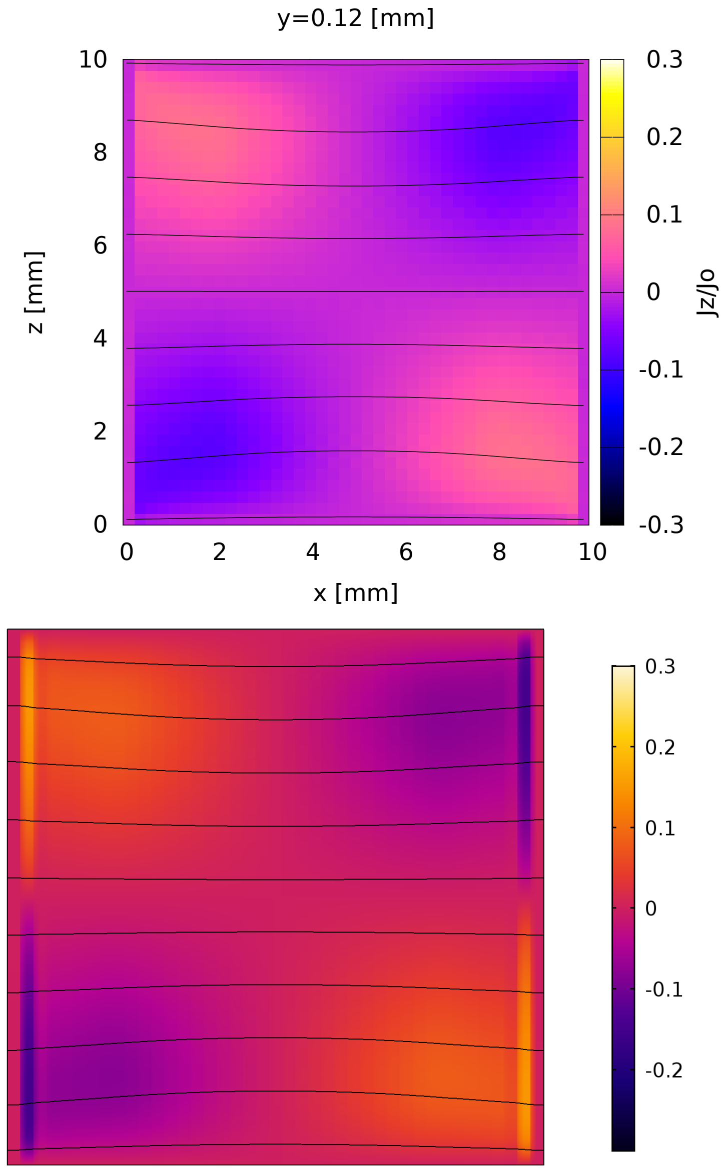

Figure 3: Component of current density J_z with current flux lines at plane y=0.12 mm.

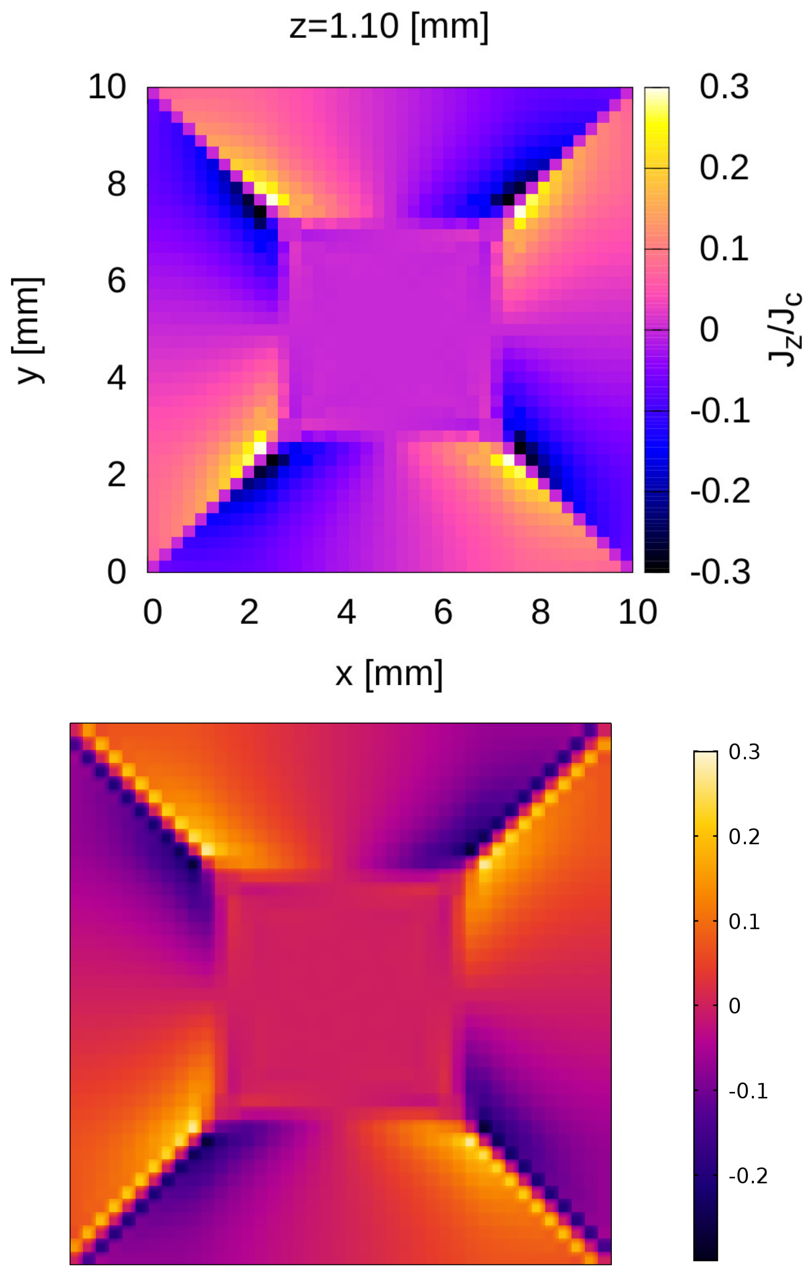

Figure 4: Component of current density J_z at plane z=1.1 mm.

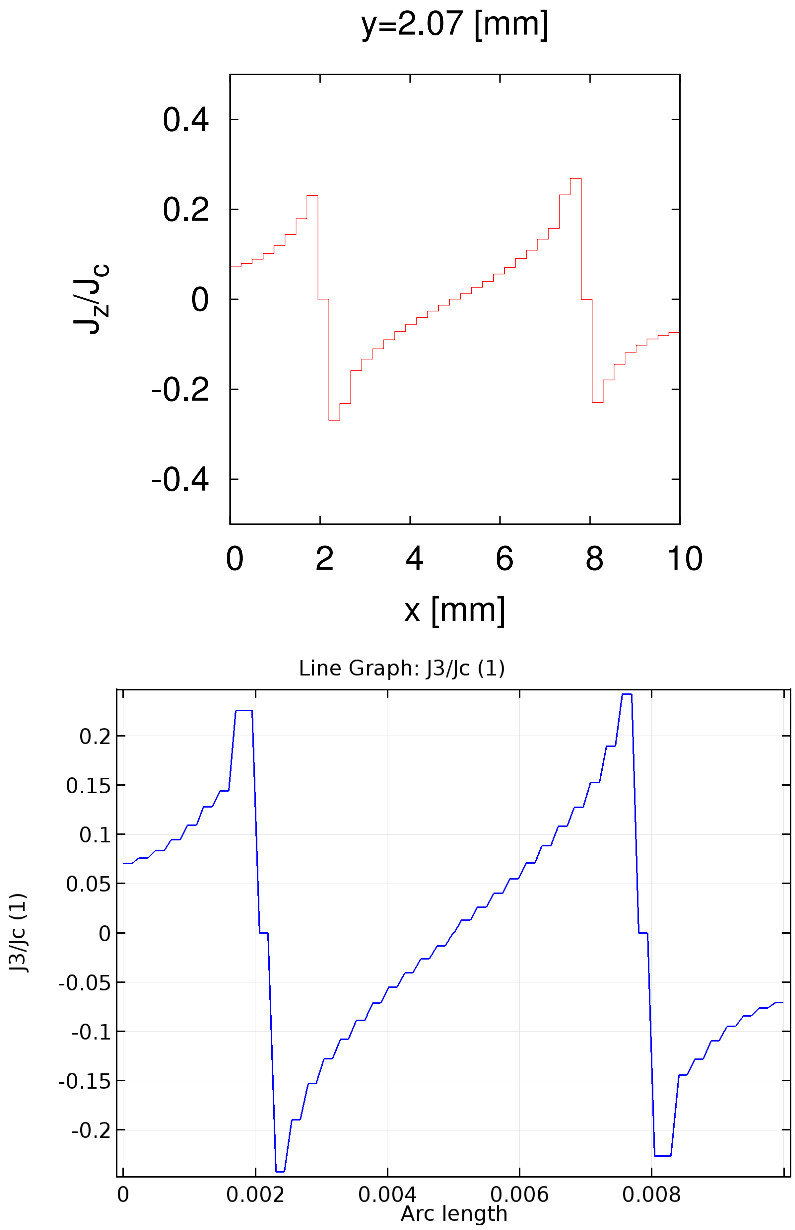

Figure 5: Component of current density J_z at plane y=2.07 mm.

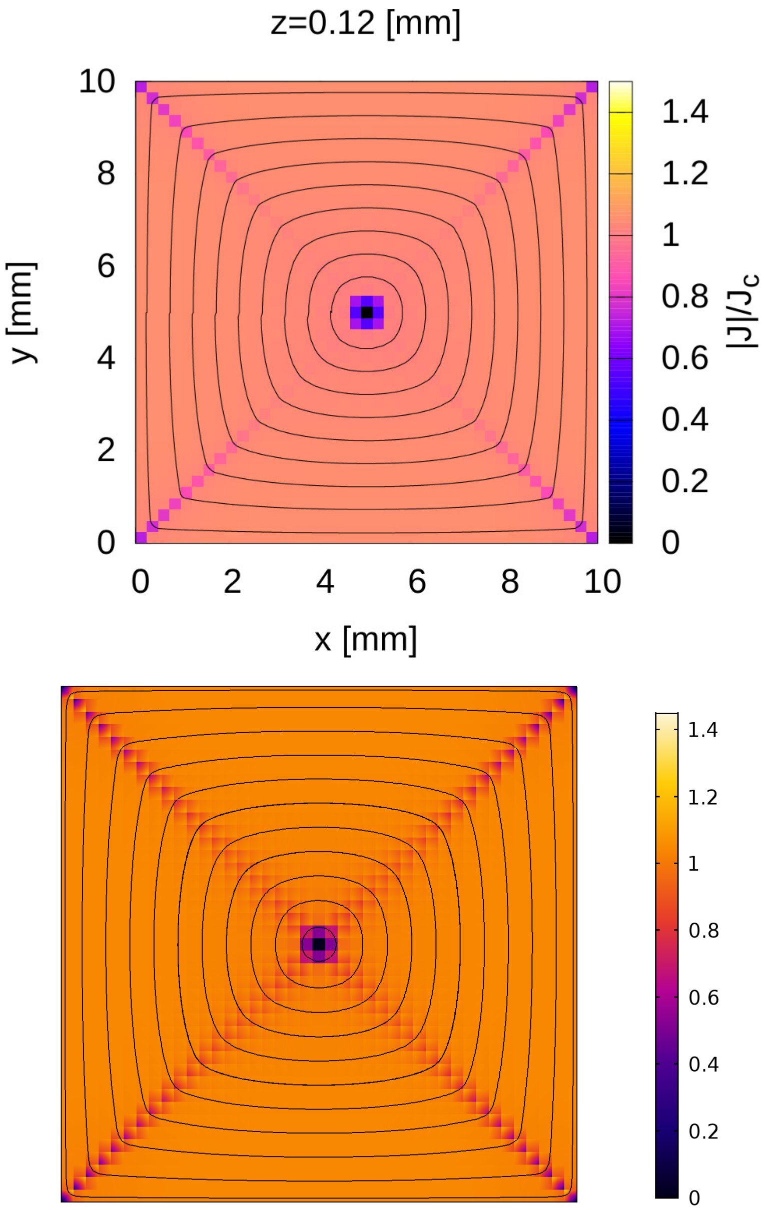

Figure 6: Modulus of current density $|\mathbf{J}|$ and current flux lines at plane z=0.12 mm.

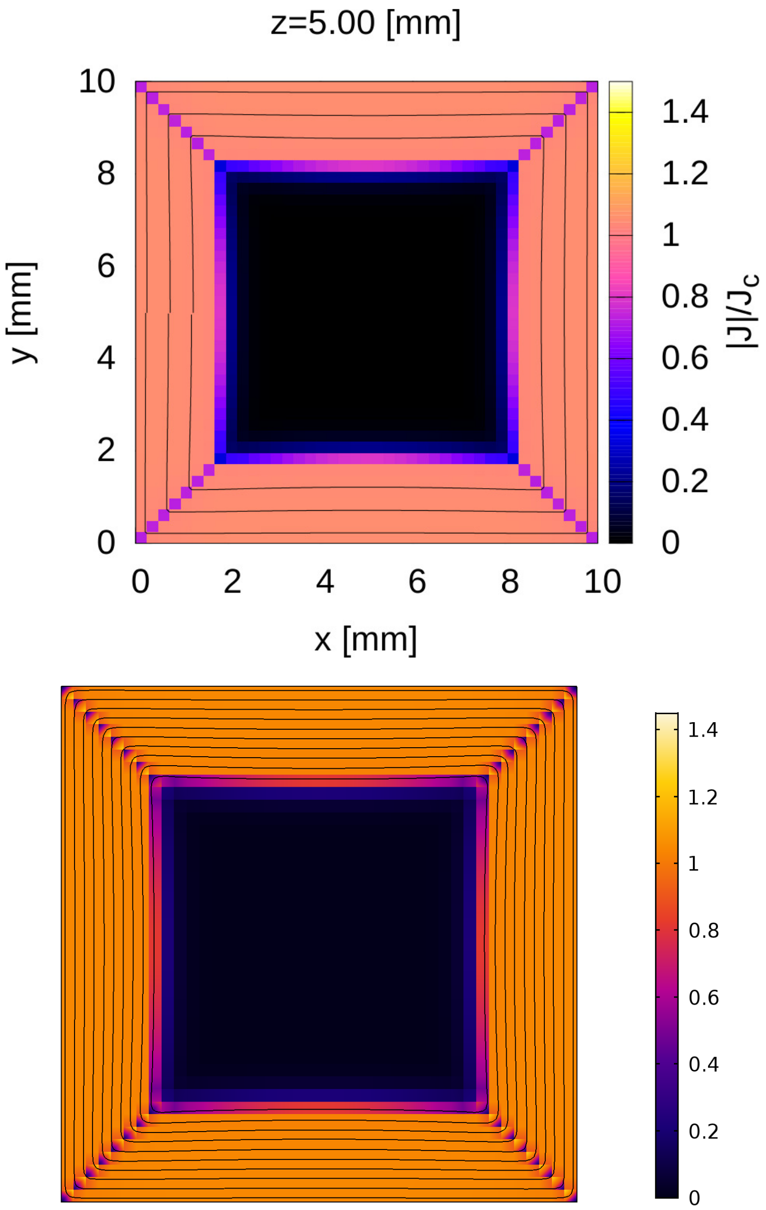

Figure 7: Modulus of current density $|\mathbf{J}|$ and current flux lines at plane z=5 mm

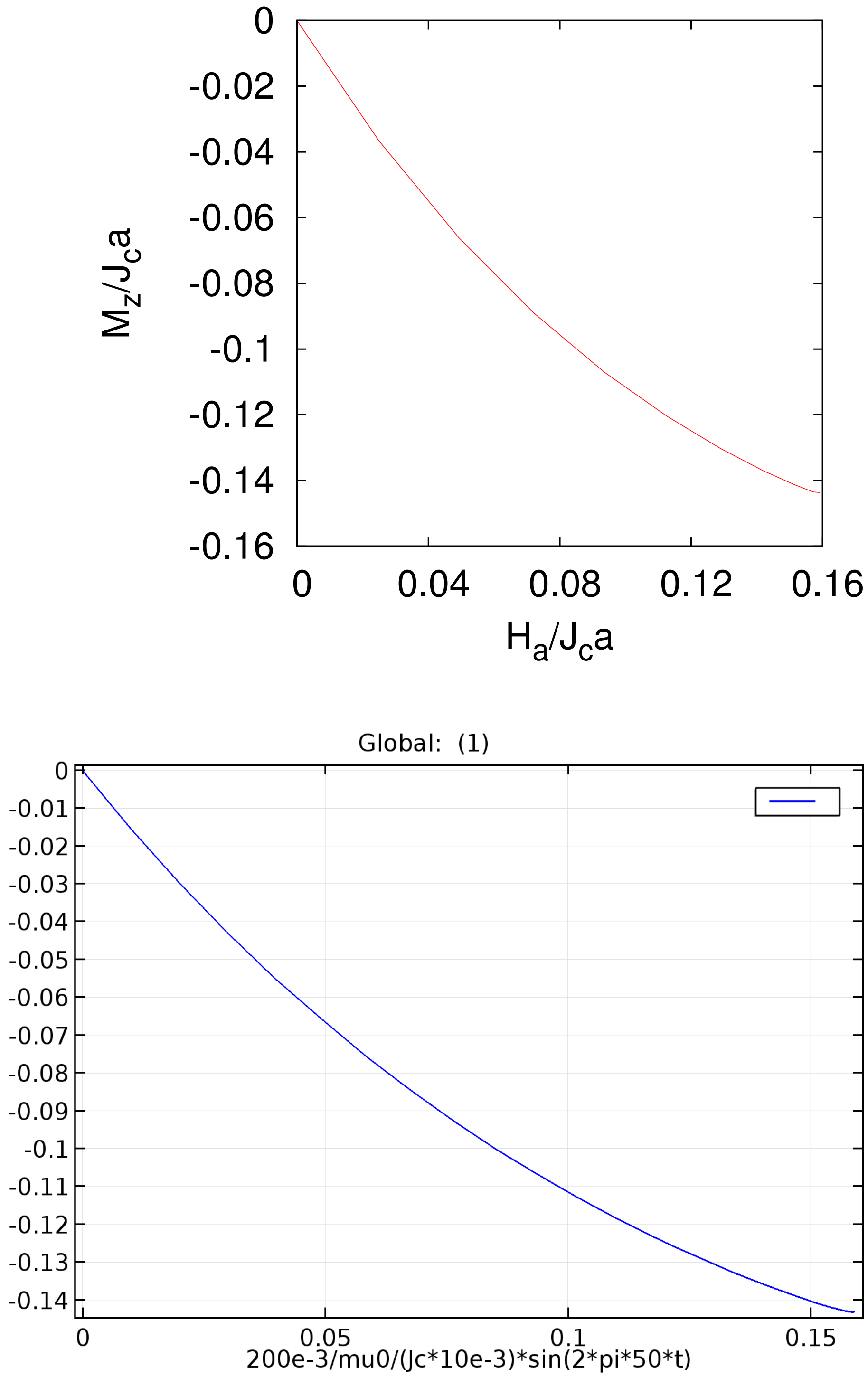

Figure 8: Magnetic moment M_z/J_ca from $B_a = 0$ to the peak.